

## Open and closed strings from unstable D-branes

Ashoke Sen\*

Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211019, India

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The tachyon effective field theory describing the dynamics of a non-BPS D- $p$ -brane has electric flux tube solutions where the electric field is at its critical value and the tachyon is at its vacuum. It has been suggested that these solutions have the interpretation of fundamental strings. We show that in order that an electric flux tube can “end” on a kink solution representing a BPS D- $(p-1)$ -brane, the electric flux must be embedded in a tubular region inside which the tachyon is finite rather than at its vacuum where it is infinite. Energetic considerations then force the transverse “area” of this tube to vanish. We suggest a possible interpretation of the original electric flux tube solutions around the tachyon vacuum as well as of tachyon matter as system of closed strings at a density far above the Hagedorn density.

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## I. INTRODUCTION

The study of various aspects of tachyon dynamics on a non-Bogomol’nyi-Prasad-Sommerfield (BPS) D- $p$ -brane of type IIA or IIB superstring theory has led to some understanding of the tachyon effective action [1–6] describing the dynamics of these branes. The bosonic part of this effective action, describing the dynamics of a tachyon field on a non-BPS D $p$ -brane of type IIA or IIB superstring theory, is given by

$$S = \int d^{p+1}x \mathcal{L},$$

$$\mathcal{L} = -V(T) \sqrt{-\det \mathbf{A}}, \quad (1.1)$$

where

$$\mathbf{A}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \partial_\mu Y^I \partial_\nu Y^I + F_{\mu\nu}, \quad (1.2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (1.3)$$

$A_\mu$  and  $Y^I$  for  $0 \leq \mu, \nu \leq p$ ,  $(p+1) \leq I \leq 9$  are the gauge and the transverse scalar fields on the world-volume of the non-BPS brane, and  $T$  is the tachyon field.  $V(T)$  is the tachyon potential which is symmetric under  $T \rightarrow -T$ , has a maximum at  $T=0$  where it is equal to the tension  $\tilde{T}_p$  of the non-BPS D- $p$ -brane, and has its minimum at  $T = \pm\infty$  where it vanishes. We are using the convention where  $\eta = \text{diag}(-1, 1, \dots, 1)$  and the fundamental string tension has been set equal to  $(2\pi)^{-1}$  (i.e.,  $\alpha' = 1$ ). References [5,7–11] suggest the choice

$$V(T) = \frac{\tilde{T}_p}{\cosh(T/\sqrt{2})}. \quad (1.4)$$

Since the tachyon has a negative mass<sup>2</sup> of the order of the string scale, the very notion of an effective action, which normally refers to the result of integrating out the heavy

modes for describing the dynamics of light modes, is somewhat unclear here. The matter is only made worse by the fact that there are no physical states around the tachyon vacuum, and hence the usual method of deriving an effective action—by comparing the  $S$ -matrix elements computed from string theory with those computed from the effective action—does not work. Thus one might wonder in what sense Eq. (1.1) describes the tachyon effective action on a non-BPS D- $p$ -brane. This question was addressed and partially answered in a recent paper [10]. As already emphasized in this paper, the usefulness of an effective action can also be judged by comparing the classical solution of the equations of motion derived from the effective action with the classical solutions in open string theory, which in turn are described by boundary conformal field theories (BCFT). In this respect the tachyon effective action given in Eq. (1.1) has had some remarkable success. Among the string theory results reproduced by this effective action are the following.

- (1) The effective action has a one-parameter family of time-dependent solutions describing the rolling of a spatially homogeneous tachyon towards the vacuum  $T = \pm\infty$ . The single parameter labeling the solution labels different initial conditions on the tachyon field which cannot be related by time translation. In full string theory, this one-parameter family of solutions can be realized as appropriate marginal deformations of the BCFT describing the original non-BPS D- $p$ -brane [12,13]. Furthermore, for the choice of  $V(T)$  given in Eq. (1.4), the time dependence of the pressure, as calculated in BCFT, resembles the result derived from the effective action [9]. However, as mentioned in [6,10,11], this resemblance is only at a superficial level. This is most easily seen by examining the energy density and pressure at the instant when the tachyon is at rest. At this instant the field theory answers for the energy density and pressures are equal in magnitude but differ by sign. No such simple relation exists for the full stringy answer. This is not necessarily a contradiction, since during the initial stages of evolution the second- and higher-derivative corrections to the action, which are not included in Eq. (1.1), may be important.

\*Email address: ashoke.sen@cern.ch; sen@mri.ernet.in

Surprisingly however, in the limit where the tachyon begins rolling from the top of the potential, the effective action (1.1)–(1.3) with potential (1.4) correctly reproduces the time evolution of the stress tensor [9,10]. The agreement between the results derived from the effective field theory action (1.1)–(1.4) and the full tree level stringy results continue to hold even in the presence of a uniform background electric field [14–16].

- (2) The effective action correctly gives the mass of the tachyon on the non-BPS D- $p$ -brane if we choose  $V(T)$  as in Eq. (1.4). In the language of classical solutions, this can be restated by saying that it correctly reproduces the solution of the linearized equations of motion for the tachyon (and the massless fields) around the  $T=0$  configuration.
- (3) The effective action has a kink solution of zero width [17–23] representing a BPS D- $(p-1)$ -brane [24–26]. Furthermore, the world-volume action on the kink coincides with that on a BPS D- $(p-1)$ -brane [22,27–29]. (See also Refs. [30,31].) If we choose the potential as in Eq. (1.1), the tension of the kink also agrees with the tension of the D- $(p-1)$ -brane [9].
- (4) If we compactify one of the directions on the original D- $p$ -brane on a circle of radius  $R$ , then at a critical radius  $R=\sqrt{2}$ , the BCFT describing the non-BPS D- $p$ -brane admits a marginal deformation which smoothly interpolates between a non-BPS D- $p$ -brane and a BPS D- $(p-1)$ -brane– $\bar{D}$ - $(p-1)$ -brane pair situated at diametrically opposite points on the circle [25]. It turns out that for the choice of potential given in Eq. (1.4), the effective action (1.1)–(1.3) correctly reproduces this property [9]. Namely, if we compactify one of the spacelike coordinates on a circle of radius  $\sqrt{2}$ , then the equations of motion admit a one-parameter ( $a$ ) family of solutions, such that at one end of the parameter space ( $a=0$ ) we have the configuration  $T=0$  representing the original non-BPS D- $p$ -brane, while at the other end of the parameter space ( $a=\infty$ ) we have a kink-antikink pair situated at diametrically opposite points on the circle, representing a D- $(p-1)$ -brane– $\bar{D}$ - $(p-1)$ -brane pair.
- (5) If we consider an inhomogeneous time-dependent solution by choosing the initial condition  $T=T_0 \sin x$ ,  $\dot{T}=0$  and let the tachyon evolve according to the equations of motion derived from the effective action (1.1), then the solution hits a singularity after a *finite time interval* at the points  $x=n\pi$  for integer  $n$  [20]. One could ask if this is also a feature of the corresponding BCFT. Unfortunately the BCFT describing this situation is not exactly solvable, but the corresponding problem for D- $p$ -branes in bosonic string theory is exactly solvable and displays precisely the feature that the energy momentum tensor blows up at isolated values of  $x$  after a finite time interval [32].
- (6) The equations of motion derived from the effective action also admit electric flux tube solutions [33–38]. These solutions are characterized by the fact that the

tachyon is at its vacuum ( $T=\infty$ ), the electric field  $\vec{E}$  takes its limiting value  $|\vec{E}|=1$ , and the electric flux is nonzero in some region of space. The spatially homogeneous version of these solutions can be realized as appropriate deformations of the BCFT describing the non-BPS D- $p$ -brane with a background electric field [14,15].

The localized electric flux tube solutions mentioned in item (6) above have been proposed as candidates for describing fundamental strings. Indeed, these solutions have many properties in common with the fundamental string, including the quantum numbers and dynamics [33–35]. However, these solutions also suffer from the difficulty that the flux can spread out in the transverse directions instead of being confined into a narrow tube [33,34,39]. This will correspond to a new degree of freedom corresponding to fattening of the fundamental string and is contrary to the known property of the fundamental string.

In this paper we show that if an electric flux tube has one of its ends “attached” to a kink, then as we move a distance  $x$  away from the kink along the flux tube, the tachyon cannot increase faster than  $x^{1/2}$ . This result, in turn, can be used to argue that the usual exchange interaction, expected of a fundamental string, is suppressed for these flux tubes. Hence we have another reason for not using these solutions to describe a fundamental string. In order to overcome this problem, we propose another class of solutions as candidates for fundamental strings based on the result of [40–43]. These solutions have the right quantum numbers and dynamics as a fundamental string, have the ability to end on a kink, and can also have the usual exchange interaction. The construction of these solutions requires creating a core where the tachyon solution is away from its vacuum value and the electric flux is embedded in this core. Energetic considerations then force this core to have zero “area” in the hyperplane transverse to the flux. Consequently, the electric flux is also confined to a region of zero volume. This property is consistent with that of the fundamental string, but this is only a partial success, as confinement of the flux within a region of zero volume does not necessarily imply confinement to a one-dimensional subspace as would be required if it has to describe a fundamental string. We suggest a possible resolution of this puzzle based on higher-derivative corrections on the D-brane world volume.

In this context we also recall that in several recent papers precisely this type of configuration involving electric flux has been considered from another viewpoint [39,44,45]. In these papers the authors studied the process of inhomogeneous tachyon condensation on an unstable D-brane in the presence of an electric field and argued that at the end of the condensation process the electric flux gets confined to regions inside which the tachyon is finite, rather than being spread out into the fat flux tube solutions described in [34] for which the tachyon is at its vacuum everywhere. Based on this analysis the authors argued that fundamental string solutions must be described by the former type of solutions. Although we arrive at these solutions from a different point of view, our final conclusion agrees with

that of Refs. [39,44,45].<sup>1</sup>

These results still leave open the question, what is the physical interpretation of the original electric flux tube solutions of [33–38] inside which the tachyon is at its vacuum everywhere? A similar question can be asked about the tachyon matter solution of [12,13,5,20,21,32,46–61]. We suggest a possible interpretation of these solutions as a system of high-density closed string states. Our arguments rely on the results of [9,62] where the authors argued that during the decay of a D-brane all the energy of the brane is converted into closed strings. Naively one might expect that this invalidates the open string analysis of [12,13]. However, since closed strings are automatically included in quantum open string theory, one could argue that the effect of emission of closed strings should be included in quantum open string theory and need not have to be included as a separate effect. Thus the closed string description of the D-brane decay should be equivalent to the quantum open string description rather than being a replacement of the latter. The results in quantum open string theory, in turn, should reduce to those in the classical open string theory in the weak string coupling limit. For consistency, this requires that the properties of the system of closed strings produced in the decay of a D-brane must agree with the results of classical open string theory in the weak string coupling limit. We show that this is indeed the case. This suggests that in the weak coupling limit the classical tachyon matter described in [12,13] gives a description of a system of closed strings at density of order  $g^{-1}$ : i.e., far above the Hagedorn density. Generalizing this, we also propose that the electric flux tube solutions with the electric field directed along a compact direction describes a system of closed strings at high energy density and high density of fundamental string winding charge. This indicates that in general the solutions in the classical open string theory (or tachyon effective field theory) around the tachyon vacuum where  $T$  is large everywhere give an effective description of closed strings at a density of order  $g^{-1}$ . An interpretation of tachyon matter in a somewhat similar spirit has been discussed independently in [62].

## II. REVIEW OF KINK AND FLUX TUBE SOLUTIONS

We begin by reviewing the flux tube solutions of Refs. [33–36]. These are most easily seen in the Hamiltonian formalism given in [34]. We denote by  $\Pi^i$  the momenta conjugate to the gauge field components  $A_i$  ( $1 \leq i \leq p$ ), by  $P_I$  the momenta conjugate to the scalar field  $Y^I$ , and by  $\Pi^T$  the momentum conjugate to the tachyon field  $T$ :

<sup>1</sup>We should note, however, that the explicit string theoretic analysis based on boundary state has been carried out only in the context of bosonic string theory [39], since a perturbation describing inhomogeneous rolling tachyon gives rise to a solvable BCFT only in this case [32]. Thus these results are not directly applicable in the present context where we focus our attention on the superstring theory.

$$\begin{aligned}\Pi^i &= \frac{\delta S}{\delta(\partial_0 A_i)} = V(T)(\mathbf{A}^{-1})_A^{i0} \sqrt{-\det \mathbf{A}}, \\ P_I &= \frac{\delta S}{\delta(\partial_0 Y^I)} = V(T)(\mathbf{A}^{-1})_S^{0\nu} \partial_\nu Y^I \sqrt{-\det \mathbf{A}}, \\ \Pi_T &= \frac{\delta S}{\delta(\partial_0 T)} = V(T)(\mathbf{A}^{-1})_S^{0\nu} \partial_\nu T \sqrt{-\det \mathbf{A}},\end{aligned}\quad (2.1)$$

where the subscripts  $S$  and  $A$  denote the symmetric and antisymmetric components of a matrix, respectively. Then the Hamiltonian is given by

$$\begin{aligned}H &= \int d^p x \mathcal{H}(x), \quad \mathcal{H}(x) = \sqrt{\mathcal{K}(x)}, \\ \mathcal{K}(x) &= \Pi^i \Pi^i + P_I P_I + \Pi_T \Pi_T + (\Pi^i \partial_i Y^I)(\Pi^j \partial_j Y^I) \\ &\quad + (\Pi^i \partial_i T)(\Pi^j \partial_j T) + (F_{ij} \Pi^j + \partial_i Y^I P_I + \partial_i T \Pi_T) \\ &\quad \times (F_{ik} \Pi^k + \partial_i Y^I P_J + \partial_i T \Pi_T) + V^2 \det(h), \\ h_{ij} &= \delta_{ij} + F_{ij} + \partial_i Y^I \partial_j Y^I + \partial_i T \partial_j T.\end{aligned}\quad (2.2)$$

The equations of motion derived from this Hamiltonian need to be supplemented by the Gauss' law constraint

$$\partial_i \Pi^i = 0. \quad (2.3)$$

Around the minimum of the tachyon potential at  $T = \infty$  the theory contains solutions describing electric flux tubes. For example, the configuration

$$\Pi^1(x) = f(x^2, \dots, x^p), \quad A_1(x) = x^0, \quad T = \infty, \quad (2.4)$$

with all other fields and their conjugate momenta set to zero, can be shown to be a solution of the equations of motion and describes electric flux along the  $x^1$  direction.  $f(x^2, \dots, x^p)$  is an arbitrary positive semidefinite function of the coordinates transverse to the direction of the flux. The total flux or fundamental string charge associated with this configuration is given by

$$\mathcal{F} = \int dx^2 \cdots dx^p f(x^2, \dots, x^p). \quad (2.5)$$

This is quantized in units of fundamental string charge. Taking  $f$  to be a delta function in these transverse coordinates gives a stringlike configuration whose dynamics agrees with that of the fundamental string [34,35,38], but clearly the freedom of choosing an arbitrary  $f$  shows that the flux can spread out, unlike that of a fundamental string which has zero width.

As already stated, the equations of motion derived from the Hamiltonian (2.2) also have a tachyon kink solution [17–22]. It has the property that it has zero width and interpolates between  $T = -\infty$  for  $x < 0$  to  $T = \infty$  for  $x > 0$ . For describing

electric flux in the presence of a kink, it is useful to take the kink as the  $a \rightarrow \infty$  limit of the configuration [19]

$$T(x^p) = aF(x^p), \quad (2.6)$$

where  $F(x)$  satisfies

$$\begin{aligned} F(-x) &= -F(x), \quad F'(x) > 0 \quad \text{for} \quad -b < x < b, \\ F'(\pm b) &= 0, \quad F(x) = F(b) \quad \text{for} \quad x \geq b, \\ F(x) &= F(-b) \quad \text{for} \quad x \leq -b, \end{aligned} \quad (2.7)$$

$b$  being a constant which can be taken to be as small as we like *after we have taken the  $a \rightarrow \infty$  limit*. This solution has the property that  $\partial_i T$  goes to zero outside the range  $-b < x^p < b$ , but in the range  $-b < x^p < b$   $|\partial_{x^p} T|$  is infinite in the  $a \rightarrow \infty$  limit, and  $T(x^p)$  interpolates between  $T(x^p) = -\infty$  for  $x^p < 0$  and  $T(x^p) = \infty$  for  $x^p > 0$ .

This description is slightly different from the one used in [22], but all the properties of the solution discussed in [22] remain valid with this new description. Let  $\xi \equiv (\xi^0, \dots, \xi^{p-1}) = (x^0, \dots, x^{p-1})$  denote the world-volume coordinates on the kink. As was shown in [22], the world-volume theory on the kink is described precisely by the Dirac-Born-Infeld (DBI) action involving massless scalar fields  $y^i(\xi)$  ( $p \leq i \leq 9$ ) and gauge fields  $a_\alpha(\xi)$  [ $0 \leq \alpha \leq (p-1)$ ] under the identification

$$A_p(x^p, \xi) = \phi(x^p, \xi), \quad A_\alpha(x^p, \xi) = a_\alpha(\xi) - \phi(x^p, \xi) \frac{\partial y^p(\xi)}{\partial \xi^\alpha},$$

$$\begin{aligned} T(x^p, \xi) &= aF[x^p - y^p(\xi)], \quad Y^I(x^p, \xi) = y^I(\xi), \\ \text{for } (p+1) &\leq I \leq 9, \end{aligned} \quad (2.8)$$

where  $\phi(x^p, \xi)$  is an arbitrary smooth function. In fact  $A_\mu(x^p, \xi)$  [ $Y^I(x^p, \xi)$ ] can be taken to be any smooth vector [scalar] field whose pullback along the kink world volume  $x^p = y^p(\xi)$  is equal to  $a_\alpha(\xi)$  [ $y^I(\xi)$ ]. Using these relations one can show that

$$\begin{aligned} \Pi^s(x^p, \xi) &= (\mathcal{T}_{p-1})^{-1} aF'[x^p - y^p(\xi)] \\ &\quad \times V\{aF[x^p - y^p(\xi)]\} \pi^s(\xi) \\ &= (\mathcal{T}_{p-1})^{-1} \pi^s(\xi) \frac{\partial}{\partial x^p} U\{T[x^p - y^p(\xi)]\}, \\ 1 &\leq s \leq (p-1), \end{aligned}$$

$$U(T) \equiv \int_{-\infty}^T V(y) dy,$$

$$\begin{aligned} \Pi^p(x^p, \xi) &= \Pi^s(x^p, \xi) \frac{\partial y^p(\xi)}{\partial \xi^s} \\ &= -(\mathcal{T}_{p-1})^{-1} \pi^s(\xi) \frac{\partial}{\partial \xi^s} U\{T[x^p - y^p(\xi)]\}, \end{aligned}$$

(2.9) Thus,

where  $\pi^s = \delta S / \delta (\partial_0 a_s)$  denotes the momenta conjugate to the gauge fields on the kink world volume and

$$\mathcal{T}_{p-1} = \int_{-\infty}^{\infty} V(T) dT = U(\infty) \quad (2.10)$$

is the tension of the kink. Since  $V(T)$  falls off exponentially for large  $T$ , we see that the most of the contribution to  $\Pi^s$  comes from the region where  $F[x^p - y^p(\xi)]$  is of order  $1/a$ : i.e., from regions of width  $\sim 1/a$ .

Incidentally, Eqs. (2.9) can also be derived from an energy minimization principle. Consider, for example, a flat BPS D- $(p-1)$ -brane at  $x^p = 0$ , with a uniform electric flux  $\pi^1$  along the  $x^1$  direction. In the tachyon effective field theory on a non-BPS D $p$ -brane, we could try to represent this by the tachyon background given in Eq. (2.6), with an  $x^p$ -dependent electric flux  $\Pi^1(x^p)$  along the  $x^1$  direction.  $\pi^1$  is related to  $\Pi^1(x^p)$  as

$$\pi^1 = \int dx^p \Pi^1(x^p). \quad (2.11)$$

From Eq. (2.2) we now see that in the  $a \rightarrow \infty$  limit the energy density associated with this configuration is given by

$$\mathcal{E} = \int dx^p \sqrt{\{\Pi^1(x^p)\}^2 + \{V(aF(x^p))aF'(x^p)\}^2}. \quad (2.12)$$

We want to find what  $\Pi^1(x^p)$ , subject to the constraint (2.11), minimizes this energy density. For this we take the ansatz

$$\Pi^1(x^p) = G'(x^p), \quad (2.13)$$

where  $G$  is some function to be determined. Equation (2.11) now gives

$$G(\infty) - G(-\infty) = \pi^1. \quad (2.14)$$

On the other hand, Eq. (2.12) gives

$$\mathcal{E} = \int dx^p \sqrt{\{G'(x^p)\}^2 + \{V(aF(x^p))aF'(x^p)\}^2}. \quad (2.15)$$

We now minimize Eq. (2.15) with respect to  $G(x^p)$ , keeping the boundary values of  $G(x^p)$  fixed. This gives

$$\frac{\partial}{\partial x^p} \left( \frac{G'(x^p)}{\sqrt{\{G'(x^p)\}^2 + \{V(aF(x^p))aF'(x^p)\}^2}} \right) = 0. \quad (2.16)$$

$$G'(x^p) = CV(aF(x^p))aF'(x^p), \quad (2.17)$$

where  $C$  is a constant. Integrating both sides over  $x^p$ , and using Eqs. (2.10), (2.14), we get

$$\pi^1 = C\mathcal{T}_{p-1}. \quad (2.18)$$

Equations (2.13), (2.17), (2.18) now give

$$\Pi^1(x^p) = (\mathcal{T}_{p-1})^{-1}aF'(x^p)V(aF(x^p))\pi^1. \quad (2.19)$$

Equation (2.19) is a special case [ $y^p(\xi) = 0$ ] of the more general result given in Eqs. (2.9).

### III. NO GO THEOREM

Since a kink solution in this theory is expected to describe a D- $(p-1)$ -brane, we would expect that fundamental strings should be able to end on this D- $(p-1)$ -brane. Since the end of the fundamental string carries electric charge under the  $U(1)$  gauge field residing on the D- $(p-1)$ -brane, this will give rise to a gauge field background on the D- $(p-1)$ -brane. According to Eqs. (2.9) this implies that the electric flux associated with the fundamental string should be able to penetrate to the core of the kink where  $F[x^p - y^p(\xi)] \sim a^{-1}$  and  $T = aF[x^p - y^p(\xi)]$  is finite. We shall now show that there are strong constraints on finding electric flux tube solutions of this type.

We begin by noting that  $\mathcal{K}(x)$  given in Eq. (2.2) is a sum over a set of terms each of which is positive semidefinite. This allows us to put a lower bound to the total energy associated with any configuration as follows:

$$\begin{aligned} E = H &\geq \int d^p x \sqrt{(\Pi^i \partial_i T)(\Pi^j \partial_j T)} = \int d^p x |\Pi^i \partial_i T| \\ &= \int d^p x |\partial_i(\Pi^i T)|. \end{aligned} \quad (3.1)$$

In the last step we have used the Gauss' law constraint (2.3). Let us now evaluate the contribution to the right hand side of Eq. (3.1) from a narrow tube around an electric flux line, with the walls of the tube being parallel to the flux line, and the two ends  $A$  and  $B$  capped by disks of cross section  $d\sigma_A$  and  $d\sigma_B$  orthogonal to the flux lines. Let  $T_A$  and  $T_B$  be the values of the tachyon field at the two ends and  $\Pi_A$  and  $\Pi_B$  be the magnitudes of  $|\tilde{\Pi}|$  at the two ends. Then, since there is no leakage of flux through the wall of the tube, the Gauss' law constraint (2.3) gives

$$\Pi_A d\sigma_A = \Pi_B d\sigma_B \equiv d\mathcal{F}, \quad (3.2)$$

where  $d\mathcal{F}$  denotes the total flux flowing along the tube. On the other hand, evaluating the contribution to the right hand side of Eq. (3.1) from this tube we see that the total energy  $dE$  contained in this tube has a lower bound of the form

$$dE \geq |\Pi_A T_A d\sigma_A - \Pi_B T_B d\sigma_B| = |d\mathcal{F}(T_A - T_B)|. \quad (3.3)$$

This shows that if for a finite amount of flux flowing along a tube the value of the tachyon along a flux line changes by an

infinite amount—as will be the case if the electric flux enters the core of the kink from outside where  $T = \infty$ —then it costs an infinite amount of energy. Put another way, the electric flux tubes of the kind described in Eqs. (2.4) are repelled by the tachyon kink for  $|x| < b$ , since  $|\vec{\nabla}T|$  blows up in this range. Hence such flux tubes cannot “end” on a D-brane.

Note, however, that this argument only prevents the tachyon from changing from a finite value to  $\infty$  along a flux line *within a finite distance*. An infinite-length flux tube can have at one end a finite  $T$  and at the other end infinite  $T$ . To see how this is possible, let us take a flux line along (say) the  $x$  direction, carrying total electric flux  $\Pi$ , and let  $T(x)$  denote the tachyon profile along  $x$ . We shall take the tachyon to be large so that the term involving the tachyon potential can be ignored. Then the total integrated energy associated with the flux line is given by

$$\int dx |\Pi| \sqrt{1 + (\partial_x T)^2} \leq \int dx |\Pi(x)| \left( 1 + \frac{1}{2} (\partial_x T)^2 \right). \quad (3.4)$$

On the right hand side of Eq. (3.4) the first term  $\int dx |\Pi(x)|$  is just the energy cost due to the fundamental string tension which is always present. Thus the excess contribution to the energy due to the variation of  $T$  along the flux line is bounded from above by

$$\frac{1}{2} \int dx |\Pi| (\partial_x T)^2. \quad (3.5)$$

Now suppose  $T = T_0$  at  $x = 0$  where  $T_0$  is some arbitrary large but finite constant. If we take, e.g.,  $T$  to vary along the flux tube as

$$T(x) = T_0 - \alpha + \alpha(1+x)^\beta \quad (3.6)$$

for some constants  $\alpha$  and  $\beta$  with  $0 < \beta < \frac{1}{2}$ , then  $T$  approaches  $\infty$  as  $x \rightarrow \infty$ . On the other hand, Eq. (3.5) shows that the total energy cost for this configuration is bounded from above by

$$\frac{1}{2} |\Pi| \frac{\alpha^2 \beta^2}{1 - 2\beta}. \quad (3.7)$$

This can be made as small as we like by taking  $\alpha$  sufficiently small. Thus we see that at little cost in energy, it is possible to have configurations where the tachyon grows slowly towards infinity as we move along the flux line away from the plane of the kink. We can put a bound on how fast the tachyon can grow by requiring that the excess energy

$$T_{00}^{excess} \equiv \int_0^\infty dx |\Pi| [\sqrt{1 + (\partial_x T)^2} - 1], \quad (3.8)$$

be finite. If  $T \sim x^\beta$  for large  $x$ , then this gives a bound  $\beta < \frac{1}{2}$ , and the excess energy density behaves for large  $x$  as

$$T_{00}^{excess} \sim x^{2\beta-2}. \quad (3.9)$$

The results of this section hold also for a configuration describing a flux tube passing through the kink rather than ending on it, since in order to pass through the kink the flux must travel through a region inside which the tachyon is finite. Since there are explicit classical solutions describing electric flux passing through a kink, both as exact classical solutions in open string theory [14,15] and as classical solutions in the tachyon effective field theory [16], we shall examine these solutions in the next section and explicitly verify the various bounds derived in this section.

#### IV. BOUNDARY CONFORMAL FIELD THEORY ANALYSIS

We shall now construct, using BCFT techniques, a periodic array of kink-antikink pairs on a non-BPS  $D_p$ -brane in the presence of an electric field in direction transverse to the kink world volume and show that the results are consistent with the analysis of Sec. III. We begin by reviewing the case without the electric field. This construction requires us to switch on a tachyon field configuration of the form  $T \propto \tilde{\lambda} \cos(x/\sqrt{2})$  [24,25], where  $x$  denotes a particular direction on the  $D_p$ -brane. For definiteness we shall take  $x = x^p$ . The space-time energy-momentum tensor associated with this deformed BCFT can be obtained by examining the associated boundary state and can in fact be read out using a Wick rotation of the results in [13]:

$$T_{00} = \tilde{\mathcal{T}}_p f(x), \quad T_{pp} = -\frac{1}{2} \tilde{\mathcal{T}}_p [1 + \cos(2\pi\tilde{\lambda})],$$

$$T_{ij} = -\tilde{\mathcal{T}}_p f(x) \delta_{ij} \quad \text{for } 1 \leq i, j \leq (p-1), \quad (4.1)$$

where

$$f(x) = \frac{1}{1 + \sin^2(\pi\tilde{\lambda})e^{i\sqrt{2}x}} + \frac{1}{1 + \sin^2(\pi\tilde{\lambda})e^{-i\sqrt{2}x}} - 1$$

$$= \frac{[1 + \sin^2(\pi\tilde{\lambda})]\cos^2(\pi\tilde{\lambda})}{\cos^4(\pi\tilde{\lambda}) + 2\sin^2(\pi\tilde{\lambda})[1 + \cos(\sqrt{2}x)]}. \quad (4.2)$$

From this we see that as  $\tilde{\lambda} \rightarrow 1/2$ ,  $f(x)$  vanishes everywhere except in the neighborhood of  $x = (2n+1)\pi/\sqrt{2}$  for integer  $n$ . A close examination shows that in the  $\tilde{\lambda} \rightarrow 1/2$  limit  $T_{00}$  receives a delta-function contribution equal to

$$T_{00} = \mathcal{T}_{p-1} \sum_n \delta(x^p - (2n+1)\pi/\sqrt{2}), \quad (4.3)$$

where  $\mathcal{T}_{p-1} = \sqrt{2}\pi\tilde{\mathcal{T}}_p$  is the tension of a BPS  $D-(p-1)$ -brane. Thus at  $\tilde{\lambda} = 1/2$  the BCFT describes an

array of  $D-(p-1)$ -brane- $\bar{D}-(p-1)$ -brane pairs [25].<sup>2</sup> If we take  $\tilde{\lambda} \approx 1/2$  instead of  $\tilde{\lambda} = 1/2$ , it is easy to see from Eq. (4.2) that the energy density is concentrated in a region of width

$$\Delta \sim \cos^2(\pi\tilde{\lambda}), \quad (4.4)$$

around the points  $x = (2n+1)\pi/\sqrt{2}$ .

Let us now consider switching on an electric field  $e$  along the  $x^p \equiv x$  direction. In this case  $T \propto \tilde{\lambda} \cos(x/\sqrt{2})$  is no longer a marginal deformation, but  $T \propto \tilde{\lambda} \cos(\sqrt{1-e^2}x/\sqrt{2})$  is [14]. The energy-momentum tensor associated with the deformed BCFT can be read out from the results of [14,15] by a double Wick rotation and is given by

$$T_{00} = \frac{1}{2} \tilde{\mathcal{T}}_p \{e^2(1-e^2)^{-1/2} [1 + \cos(2\pi\tilde{\lambda})] + 2(1-e^2)^{1/2} f(\sqrt{1-e^2}x)\},$$

$$T_{pp} = -\frac{1}{2} \tilde{\mathcal{T}}_p (1-e^2)^{-1/2} [1 + \cos(2\pi\tilde{\lambda})],$$

$$T_{ij} = -(1-e^2)^{1/2} f(\sqrt{1-e^2}x) \delta_{ij} \quad \text{for } 1 \leq i, j \leq (p-1),$$

$$\Pi^p = \frac{1}{2} \tilde{\mathcal{T}}_p e(1-e^2)^{-1/2} [1 + \cos(2\pi\tilde{\lambda})]. \quad (4.5)$$

Here  $\Pi^p$  denotes the electric flux along the  $x^p$  direction or, equivalently, fundamental string charge.

If we keep  $e$  fixed and take  $\tilde{\lambda} \rightarrow 1/2$  limit, then, as in the previous case,  $T_{00}$  acquires a delta-function contribution at the points  $(2n+1)\pi/\sqrt{2}(1-e^2)$ . However, in this limit  $\Pi^p$  vanishes; thus there is no fundamental string charge left. If we want to try to construct a configuration where there is a nonzero electric flux along the  $x^p$  direction and at the same time take the  $\tilde{\lambda} \rightarrow 1/2$  limit, we must take the  $e \rightarrow 1$  limit simultaneously, holding fixed the combination

$$\Pi^p = \frac{1}{2} \tilde{\mathcal{T}}_p (1-e^2)^{-1/2} [1 + \cos(2\pi\tilde{\lambda})] = \tilde{\mathcal{T}}_p (1-e^2)^{-1/2} \cos^2(\pi\tilde{\lambda}). \quad (4.6)$$

Analyzing Eq. (4.5) we see that in this limit the first term in the expression for  $T_{00}$  is equal to  $|\Pi^p|$  and represents the contribution coming from the electric flux, whereas the second term, involving the function  $f(\sqrt{1-e^2}x)$ , goes as

$$T_{00}^{excess} \equiv T_{00} - |\Pi^p| = \frac{2|\Pi^p|}{(\Pi^p/\tilde{\mathcal{T}}_p)^2 + 2\tilde{x}^2}, \quad \tilde{x} = x - \frac{\pi}{\sqrt{2}(1-e^2)}, \quad (4.7)$$

<sup>2</sup>While the energy-momentum tensor does not distinguish a  $D-(p-1)$ -brane from a  $\bar{D}-(p-1)$ -brane, they can be distinguished by examining the expression for the Ramond-Ramond (RR) charge density.

for finite  $\tilde{x}$ . Thus the excess energy density over and above that coming from the tension of the fundamental string is no longer strictly localized at  $\tilde{x}=0$ . In particular, for large  $\tilde{x}$ ,  $T_{00}^{excess}$  falls off as  $|\Pi^p|/\tilde{x}^2$ . This is perfectly consistent with the results of Sec. III and comparing Eq. (3.9) with Eq. (4.7) we see that the BCFT results are consistent with a logarithmic growth of the tachyon at large  $\tilde{x}$ . This can be taken to be another piece of evidence that the tachyon effective action given in Eqs. (1.1)–(1.4) correctly reproduces the properties of tree level open string theory.<sup>3</sup>

In fact, if we work with the potential (1.4), then we can explicitly reproduce this logarithmic growth in the effective field theory using the explicit solutions constructed in [16]. We can obtain these solutions from those in [9] (Appendix B) by scaling the  $x$  coordinate by  $\sqrt{1-e^2}$  [14]. This gives a periodic array of kink-antikink solutions in the presence of an electric field [16]:

$$T = \sqrt{2} \sinh^{-1} \left[ a \sin \left( \sqrt{1-e^2} \frac{\tilde{x}}{\sqrt{2}} \right) \right], \quad \tilde{x} = x - \frac{\pi}{\sqrt{2(1-e^2)}}, \quad (4.8)$$

where  $a$  is a parameter labeling the solution. The associated value of  $\Pi^p$ , computed using the method of [14] on the results of [9], is given by [16]

$$\Pi^p = \tilde{T}_p e(1-e^2)^{-1/2} (1+a^2)^{-1/2}. \quad (4.9)$$

The limit we want to consider now is  $e \rightarrow 1$ ,  $a \rightarrow \infty$ , keeping fixed

$$\Pi^p \simeq \tilde{T}_p (1-e^2)^{-1/2} a^{-1}. \quad (4.10)$$

Equation (4.8) now gives, in this limit [16],

$$T = \sqrt{2} \sinh^{-1} \left( \frac{1}{\sqrt{2}} \frac{\tilde{T}_p}{|\Pi^p|} \tilde{x} \right). \quad (4.11)$$

This gives the correct logarithmic growth of  $T$  at large  $\tilde{x}$ . The associated value of  $T_{00}^{excess}$ , computed using the effective field theory, is given by [16]

$$T_{00}^{excess} \equiv T_{00} - |\Pi_p| = \frac{2|\Pi^p|}{2(\Pi^p/\tilde{T}_p)^2 + \tilde{x}^2}, \quad (4.12)$$

in qualitative agreement with the exact result (4.7). The mismatch between Eqs. (4.7) and (4.12) of course is the result of the well-known mismatch between the results of BCFT and effective field theory [6,10,11]. Note, however, that both Eqs. (4.7) and (4.12), after integration over  $\tilde{x}$ , reproduce the tension of the D- $(p-1)$ -brane. This is a reflection of the fact that both the conformal field theory and the tachyon effective

action with potential (1.4) correctly reproduce the tension of the D- $(p-1)$ -brane as a kink solution.

## V. SOLUTION REPRESENTING THE FUNDAMENTAL STRING

The analysis of Sec. III shows that for a flux tube ending on a kink, the tachyon along the flux tube cannot increase faster than  $x^{1/2}$ ,  $x$  being the distance away from the kink. The inability of the tachyon to reach its vacuum value  $\infty$  within a finite distance affects one important property—that of exchange interactions of the type expected of a fundamental string. Consider, for example, two segments of flux tubes,  $APB$  and  $DPC$ , intersecting at a point  $P$ . Let us further suppose that  $APB$  represents a segment of a flux tube at a distance  $d_1$  from a kink on which it ends and  $DPC$  represents a segment of a flux tube at a distance  $d_2$  from a different kink on which it ends. If  $d_1$  and  $d_2$  are both large but different, then the values of  $T$  inside the segments  $APB$  and  $DPC$  will also be large but different. Let us denote them by  $T_1$  and  $T_2$ , respectively.

Now consider the exchange process by which the system described above gets converted to two new segments  $APC$  and  $DPB$ . Fundamental strings are allowed to have such exchanges. However, in this case, inside  $APC$  and  $DPB$ , the tachyon must jump from  $T_1$  to  $T_2$  (or  $T_2$  to  $T_1$ ) across the point  $P$ . Since this costs energy, such processes will not be energetically favorable. Thus we see that even if we are able to construct electric flux tube solutions for which  $T$  grows as a power law along the flux tube, such solutions will be missing one important property of the fundamental string.

Using the insight gained from the analysis of Sec. III, we shall now explore the possibility of constructing a different type of stringlike solution which carries electric flux as is required of a fundamental string, can end on the kink, and can also have exchange interactions of the kind described above. In order that the string can end on a kink, the flux lines inside the string must be able to smoothly match the electric flux lines inside a kink which flow radially outwards from the point where the string ends. Since the lower bound to the energy given in Eq. (3.1) is proportional to the component of  $\vec{\nabla}T$  along the flux line, we can try to avoid this energy cost by making the flux lines follow a constant tachyon profile. If we follow this approach, then the  $\vec{\Pi}$  across a cross section of the fundamental string should be correlated with  $T$  in the same manner in which  $\vec{\Pi}$  and  $T$  are correlated inside a kink via Eq. (2.9). Since inside a D-brane most of the contribution to the electric flux comes from regions where  $T$  is finite, the same must be true for the fundamental string configurations. In other words, in order to embed a fundamental string solution in the tachyon vacuum, we should create a region of finite  $T$  and embed the electric flux in this region. Since it costs energy to create a region of finite  $T$  due to the nonzero value of the tachyon potential and the derivative of the tachyon, we must minimize the energy. This requires the volume of this region to vanish. Thus unlike the flux tube solution of Eq. (2.4), these new configurations cannot spread over a finite  $(p-1)$ -volume transverse to the direction of the flux.

<sup>3</sup>Note that if we chose to examine  $T_{\mu\nu}$  and  $\Pi^p$  for finite  $x$  instead of finite  $\tilde{x}$ , we shall get pure electric flux tube solutions of [14].

In fact an explicit construction of such a configuration which can end on the kink is already available. These are the solutions given in [40–43]. In these papers it was shown that in the presence of a point electric charge source on a D-( $p-1$ )-brane, the DBI action on the brane admits a solution which has the interpretation that the brane gets deformed into the form of a long hollow tube attached to the original brane like a spike, with the electric flux flowing along the wall of the tube. In the world-volume theory on the D-( $p-1$ )-brane the solution of Ref. [40] takes the form

$$\pi^s = \pm A \mathcal{T}_{p-1} \frac{\xi^s}{r^{p-1}}, \quad y^p = \frac{A}{p-3} \frac{1}{r^{p-3}}, \quad r = \sqrt{\sum_{s=1}^{p-1} \xi^s \xi^s}, \quad (5.1)$$

where  $A$  is a constant labeling the total amount of flux carried by the solution. The gauge field  $a_s(\xi)$  associated with this solution is determined from its equation of motion. From this we see that as  $r \rightarrow 0$ ,  $y^p \rightarrow \infty$ ; i.e., we move farther and farther away from the plane of the D-( $p-1$ )-brane ( $y^p = 0$ ). For small  $r$  the D-( $p-1$ )-brane looks like  $\mathcal{R} \times S^{p-2}$  and the radius of  $S^{p-2}$  decreases as we go away from the plane of the original D-( $p-1$ )-brane. When the constant  $A$  is adjusted so that the electric flux takes its minimum value consistent with the quantization laws, the solution was given the interpretation of a fundamental string ending on a D-( $p-1$ )-brane, and its world-volume dynamics, quantum numbers and tension were shown to be consistent with this interpretation. Since all solutions of the DBI action on a BPS D-( $p-1$ )-brane can be lifted to a solution of the tachyon effective action on a non-BPS D- $p$ -brane [22], we can now translate the solution of the DBI theory into a solution in the tachyon effective field theory. Using Eqs. (2.8), (2.9), and (5.1) we get

$$\begin{aligned} T &= aF \left( x^p - \frac{A}{p-3} \frac{1}{r^{p-3}} \right), \\ \Pi^s &= \pm A \frac{\xi^s}{r^{p-1}} aF' \left( x^p - \frac{A}{p-3} \frac{1}{r^{p-3}} \right) \\ &\quad \times V \left( aF \left( x^p - \frac{A}{p-3} \frac{1}{r^{p-3}} \right) \right), \\ \Pi^p &= \pm A^2 \frac{1}{r^{2p-4}} aF' \left( x^p - \frac{A}{p-3} \frac{1}{r^{p-3}} \right) \\ &\quad \times V \left( aF \left( x^p - \frac{A}{p-3} \frac{1}{r^{p-3}} \right) \right), \\ (\xi^0, \dots, \xi^{p-1}) &\equiv (x^0, \dots, x^{p-1}), \\ r &= \sqrt{\sum_{s=0}^{p-1} \xi^s \xi^s} = \sqrt{\sum_{s=0}^{p-1} x^s x^s}, \end{aligned} \quad (5.2)$$

and  $A_\mu$  is any smooth vector field whose pullback on the surface  $x^p = A / \{(p-3)r^{p-3}\}$  is equal to  $a_s$  computed from Eqs. (5.1).<sup>4</sup> In this solution the fundamental string corresponds to a tubular region with the wall of the tube having a thickness of order  $1/a$ . Of course we need to take the  $a \rightarrow \infty$  limit in order to ensure that Eqs. (5.2) give a solution of the equations of motion derived from the Hamiltonian (2.2). Analogous solutions in boundary string field theory have been considered earlier in Ref. [31].

This provides a description of the fundamental open string that can end on a kink. As we go away from the plane of the kink, the radius of the tube decreases. Thus the description of a fundamental string far away from the kink is given as a rolled-up kink solution with infinitesimal radius  $R$ . However, in describing the solution we need to take the  $a \rightarrow \infty$ ,  $b \rightarrow 0$ , and  $R \rightarrow 0$  limit in this specific order.<sup>5</sup> In order to see how the energetics work out, we can use the language of the world-volume theory of the D-( $p-1$ )-brane. A straight fundamental string will then be described by a D-( $p-1$ )-brane world volume of the form  $\mathcal{R} \times S^{p-2}$ , with an electric field along the direction of  $\mathcal{R}$ . If we assume for simplicity that the electric field is uniform and has magnitude  $e$  and if  $\mathcal{V}$  denotes the  $(p-2)$ -volume of  $S^{p-2}$ , then the total flux is given by  $\mathcal{T}_{p-1} \mathcal{V} e / \sqrt{1-e^2}$  and must be fixed if we are to describe a given number (say, 1) of fundamental strings. On the other hand, the energy per unit length along  $\mathcal{R}$  for this configuration is given by

$$\mathcal{E} = \mathcal{T}_{p-1} \mathcal{V} / \sqrt{1-e^2}. \quad (5.3)$$

Thus in order to get a minimum energy configuration for a given fundamental string charge, we need to minimize  $\mathcal{V} / \sqrt{1-e^2}$ , keeping  $\mathcal{V} e / \sqrt{1-e^2}$  fixed. This leads to the limit

$$\mathcal{V} \rightarrow 0, \quad e \rightarrow 1, \quad \mathcal{V} / \sqrt{1-e^2} = \text{fixed}. \quad (5.4)$$

This description makes it clear that (within this approximation) as long as we take the D-( $p-1$ )-brane to roll up in the configuration  $\mathcal{R} \times S^{p-2}$ , the fundamental open string has zero width, since requiring the volume of  $S^{p-2}$  to be 0 implies that its radius must go to zero. It is also easy to see following [34,35] that the world-volume dynamics of such a string is described by the Nambu-Goto action. For this we need to recall that the analysis of [34,35] was carried out under the assumption that the  $V^2 \det h$  term in Eqs. (2.2) can be ignored in the study of the dynamics of the flux tube, and

<sup>4</sup>Note that since there is no magnetic flux on the kink world volume, there is no topological obstruction to choosing such a smooth vector field. We could simply choose an  $A_\mu$  which has the required value on the world volume of the kink and goes to zero quickly as we move away from the location of the kink.

<sup>5</sup>These solutions may also be considered as the zero-magnetic-field limit of supertubes [63]. However, supertubes themselves, carrying electric and magnetic flux, cannot be considered as nonsingular solutions in the tachyon effective field theory, since due to the presence of the magnetic flux on the kink world volume there is now a topological obstruction to continuing the gauge field smoothly in the region inside the tube.

this was sufficient to establish that the world-volume dynamics of an infinitely thin flux tube is governed by the Nambu-Goto action without any higher-derivative corrections. Thus all we need to show is that the  $V^2 \det h$  term in Eqs. (2.2) can be neglected for studying the dynamics of flux tubes associated with a rolled-up kink solution. Now, from Eqs. (2.9) we see that the  $\Pi^s \Pi^s$  term in Eqs. (2.2) is of order  $(T_{p-1})^{-2} (aF'V)^2 \pi^s \pi^s$ . On the other hand, the  $V^2 \det h$  term in Eqs. (2.2), which is dominated by the  $\partial_i T \partial_j T$  term in  $h_{ij}$ , goes as  $V^2 (aF')^2$ . Thus in the limit (5.4), in which  $\pi^s \sim e/\sqrt{1-e^2}$  blows up, we have

$$\Pi^s \Pi^s \gg V^2 \det h. \quad (5.5)$$

Thus we can ignore the  $V^2 \det h$  term in the analysis of the dynamics of the fundamental string. This, in turn, establishes that the dynamics is governed by the Nambu-Goto action.

The above construction provides a description of the open string ending on a kink. Given this description of the fundamental open string solution, fundamental closed strings can be described as closed loops of such rolled-up kink solutions. When two such (open or closed) strings cross they can have the usual interaction in which two segments *APB* and *DPC* of fundamental string, crossing at a point *P*, can become another pair of segments *APC* and *DPB*. This is possible because the profiles of the tachyons across a cross section of the tubes representing *APB* and *DPC* are identical, except possibly a small difference in the radii of the tubes if they represent segments of open strings which are at different distances from the kinks on which they end. In contrast, if a segment *APB* of an electric flux tube in whose core  $T=\infty$  crosses a segment *DPC* of a rolled-up kink solution, then they cannot have this type of exchange interaction, since the would-be final configurations *APC* and *DPB* will involve electric flux traveling from the  $T=\infty$  vacuum to finite- $T$  region and are energetically unfavorable. This shows that it is inconsistent to identify the closed strings as the electric flux tube solutions with  $T=\infty$  core if we have identified the open strings as rolled-up kink solutions.

Before concluding this section we would like to add a word of caution. In describing the fundamental string as a rolled-up kink solution, we could in principle replace  $S^{p-2}$  by another compact  $(p-2)$ -dimensional space  $K_{p-2}$  whose volume vanishes, but which nevertheless has some dimensions finite. An example of such a compact space could be simply an elongated sphere (ellipsoid) of the form  $R^2(x^1)^2 + (x^2)^2 + \dots + (x^{p-1})^2 = R^2$ , and we take the  $R \rightarrow 0$  limit. The energetic considerations do not prevent us from having such a configuration, and this will describe a configuration in which the charge of a fundamental string along  $x^p$  spreads over the  $x^1$  direction. This clearly violates the known property of the fundamental string. We believe the resolution of this puzzle must come from taking into account higher-derivative corrections and quantum corrections to the action (1.1), since the situation that we are describing now is simply that of a rolled-up BPS  $D-(p-1)$ -brane with electric flux, and there must be an underlying mechanism that prevents the brane from collapsing in a manner that allows us to spread out a single unit of electric flux over a subspace of dimension

$>1$ . To this end note that the configuration that we are considering is far outside the domain of validity of the effective action (1.1), and only the BPS nature of the configuration can guarantee that the solution survives higher-derivative and quantum corrections. Requiring the configuration to be BPS at the quantum level could certainly fix the shape of the transverse section.

## VI. CLOSED STRINGS AND DECAYING D-BRANES

Given that the electric flux tube solution given in Eqs. (2.4) cannot be used to describe a single fundamental string, one could ask what could be the possible physical interpretation of these solutions. In this section we shall propose a possible answer to this question. However, we begin our discussion by trying to find the physical interpretation of another related system—the tachyon matter produced by a rolling tachyon at late time [12,13,5]. Both the electric flux tube solution and the tachyon matter are characterized by the fact that they involve a configuration where the tachyon remains large (near its vacuum) everywhere in space.

The rolling tachyon solution describes the process of the classical decay of a brane-antibrane system or a non-BPS D-brane. Classical analysis indicates that the rolling of the tachyon on these systems produces at late time a state of nonzero energy density, concentrated on the plane of the original brane, and vanishing pressure. In particular, for the decay of a non-BPS  $Dp$ -brane, the energy density  $\mathcal{E}$  and the pressure  $p_{\parallel}$  and  $p_{\perp}$  along directions tangential and transverse to the brane, respectively, have the form

$$\mathcal{E} = \frac{C}{g} \delta(\vec{x}_{\perp}), \quad p_{\parallel}(x^0) = \frac{1}{g} f(x^0, C) \delta(\vec{x}_{\perp}), \quad p_{\perp}(x^0) = 0, \quad (6.1)$$

where  $C$  is some constant labeling the initial condition on the tachyon field,  $g$  is the string coupling,  $\vec{x}_{\perp}$  denote directions transverse to the  $Dp$ -brane world volume, and  $f(x^0, C)$  is a function computed in [12,13] which vanishes for large  $x^0$ .

The natural question to ask now is, what is the physical interpretation of this system? Naively, since there are no physical open string states around the tachyon vacuum, one would expect that the D-brane should decay into a collection of closed strings. On the other hand, since closed strings appear in the open string loop expansion [64], one would expect that the effect of the emission of closed strings should already be contained in the quantum open string theory, and one should not have to include the effect of closed string emission as an additional contribution beyond what quantum open string theory gives us. Keeping this in view let us now try to see how quantum open string theory will modify the classical results (6.1) and then try to compare these with the expected answer that we get assuming that the unstable D-brane system decays to closed strings.

According to the Ehrenfest theorem, the classical evolution of the energy momentum tensor given in Eqs. (6.1) should reflect the evolution of the expectation value of the energy-momentum tensor in the zero-coupling limit. In other words, during the decay of a D-brane the quantum expecta-

tion values of energy density and pressure should follow this classical answer in the weak coupling limit. However, for a finite coupling quantum effects will modify these classical results by modifying the effective action and, hence, modifying the effective equations of motion. From general considerations we should then expect the quantum-corrected results for the evolution of  $\mathcal{E}$ ,  $p_{\parallel}$ , and  $p_{\perp}$  to be of the form

$$\begin{aligned}\mathcal{E} &= \frac{1}{g} \varepsilon(\vec{x}_{\perp}, x^0, g, C), \quad p_{\parallel}(x^0) = \frac{1}{g} \phi_{\parallel}(\vec{x}_{\perp}, x^0, g, C), \\ p_{\perp}(x^0) &= \frac{1}{g} \phi_{\perp}(\vec{x}_{\perp}, x^0, g, C),\end{aligned}\quad (6.2)$$

where  $\varepsilon$  and  $\phi$  are functions which are in principle computable by quantizing the theory in the rolling tachyon background. The precise form of the functions  $\varepsilon$  and  $\phi$  may depend on the choice of the “vacuum state” used for this computation since there is no natural choice of vacuum state for the time-dependent background and we have many different quantum states corresponding to the same classical configuration. However, in the  $g \rightarrow 0$  limit, we must have

$$\begin{aligned}\varepsilon(\vec{x}_{\perp}, x^0, g, C) &\rightarrow C \delta(\vec{x}_{\perp}), \quad \phi_{\parallel}(\vec{x}_{\perp}, x^0, g, C) \rightarrow f(x^0) \delta(\vec{x}_{\perp}), \\ \phi_{\perp}(\vec{x}_{\perp}, x^0, g, C) &\rightarrow 0.\end{aligned}\quad (6.3)$$

Let us now compute the expected contribution to the energy-momentum tensor associated with the closed strings to which the D-brane should decay and compare this with Eqs. (6.2), (6.3).<sup>6</sup> The computation of closed string emission from unstable D-branes has been carried out in detail recently [9,62] where it was found that for D- $p$ -branes for  $p \leq 2$ , the emission of closed strings from this background extracts all the energy of the original brane into closed string modes.<sup>7</sup> In particular, the final state for the decay of a non-BPS D0-branes is dominated by highly nonrelativistic closed strings of mass  $\sim g^{-1}$  and velocity of order  $g^{1/2}$ . Although for non-BPS D- $p$ -branes for  $p \geq 2$  naive analysis involving homogeneous rolling tachyon tells us that only a small fraction (of order  $g$ ) of the D-brane energy is radiated away into closed strings, it was argued in [9] that in the presence of any inhomogeneity, the decay of any D- $p$ -brane for  $p \geq 1$  can be thought of as the result of the decay of a collection of non-BPS D0-branes. Hence its final state will also be dominated by highly nonrelativistic closed strings of mass  $\sim g^{-1}$  and velocity of order  $g^{1/2}$ . Even for a spatially homogeneous configuration, it was shown in [62] that higher moments of

<sup>6</sup>Of course, since the closed string sector includes gravity, the definition of the stress tensor has the usual problem. However, since only a very small fraction of the energy goes into graviton states, we could consider the contribution to the energy-momentum tensor from the nongravitational sector of the closed string.

<sup>7</sup>This analysis only deals with the closed string states created from the vacuum by spacelike oscillators. There may also be interesting information in the closed string states associated with timelike oscillators [50], but we shall ignore them in the present discussion.

the energy density of the final state closed strings diverge in the  $g \rightarrow 0$  limit, although the mean value is finite. Thus there is a large uncertainty in the energy of the emitted closed strings, and presumably, once quantum corrections are included, one would find that even a homogeneous configuration decays to closed strings of energy of order  $g^{-1}$ .<sup>8</sup>

Since the closed strings produced during the decay of the non-BPS D $p$ -brane have velocity of order  $g^{1/2}$ , it takes a time of order  $g^{-1/2}$  for these closed strings to carry energy away from the original location of the brane. This is perfectly consistent with Eqs. (6.2), (6.3). In particular a specific choice of  $\varepsilon(\vec{x}_{\perp}, x^0, g, C)$  which satisfies Eqs. (6.3) is  $C[\pi/g(x^0)^2]^{n_{\perp}/2} \exp[-\vec{x}_{\perp}^2/g(x^0)^2]$  where  $n_{\perp}$  is the number of transverse dimensions. In this case for any finite  $x^0$  as we take the  $g \rightarrow 0$  limit we shall see the energy density localized on the plane of the original brane, whereas over a period of order  $g^{-1/2}$  it disperses to a distance of order 1 in the transverse directions.

In fact, the actual rate of dispersal of the energy away from the plane of the brane may be even slower due to the gravitational attraction that tends to pull the decay products towards the plane of the brane. Since Newton’s constant is of order  $g^2$ , for two objects of mass  $\sim g^{-1}$  separated by a distance of order 1, the escape velocity is of order  $g^{1/2}$ . However, since in the present case the decay products are initially localized within a smaller distance from the plane of the brane, the escape velocity will be larger than  $g^{1/2}$ . Thus the decay products, with a typical velocity of order  $g^{1/2}$ , will not be able to escape to infinity and will be pulled back towards the plane of the brane.

This argument shows that the massive decay products by themselves cannot carry the energy away from the plane of the brane. But one might expect that these very massive closed strings will eventually decay into massless states which carry energy away from the plane of the brane. Note, however, that due to the exponentially growing density of states at a high-mass level, a very massive string state will decay predominantly to other very massive string states unless such decay processes are suppressed by exponentially small matrix elements. As a result of the presence of a large number of such massive closed strings near the plane of the original brane, these strings will collide frequently, producing (predominantly) massive closed string states. Thus the tachyon matter, describing the decay product of a non-BPS D-brane, may be longer lived than one would naively expect it to be. (Incidentally, it will be interesting to see if this argument can be sharpened to estimate the lifetime of massive black holes represented by elementary string states [65–70].)

Let us now turn to an analysis of the pressure. For a collection of nonrelativistic particles, the ratio of the pressure to the energy density is proportional to the square of the average velocity of the particles. As mentioned in the previous paragraph, for the closed strings produced in the decay of non-BPS D-branes this is of order  $g$ . Thus, after all the

<sup>8</sup>I would like to thank L. Rastelli for a discussion of this point.

energy of the D0-brane has been converted to the closed string states, the pressure of the system will be of order unity, since the energy density is of order  $g^{-1}$ . This agrees perfectly with the result of Eqs. (6.2), (6.3) which states that, asymptotically, the order  $1/g$  contribution to the pressure vanishes since  $f(x^0) \rightarrow 0$  as  $x^0 \rightarrow \infty$ .

This suggests that the classical tachyon matter, produced during the decay of an unstable D-brane, may be the open string description of a collection of highly nonrelativistic closed strings of high density that is expected to be produced in this decay. Given that closed strings appear at the open string loop level, it may seem somewhat surprising that tree level open string theory contains information about the properties of closed strings. However, this could be a reflection of the fact that since there are no open string states around the tachyon vacuum, even the classical open string theory must know something about closed strings whose average property it is supposed to reproduce in the weak coupling limit [62]. This interpretation is consistent with the idea that quantization of open string theory around the tachyon vacuum should give rise to closed string theory [71–75].

One of the lessons we can learn from this interpretation is that the classical results are quite unreliable when the energy density of the system falls below the string density. In particular classical analysis tells us that the ratio of pressure to energy density vanishes even at an energy density below the string scale, but this is not expected to happen in the quantum theory since below the string density the system should behave as ordinary radiation. From the general form (6.1) we see that this happens because in this case quantum corrections, which are of order 1, could dominate the classical contributions to  $T_{\mu\nu}$ .

Let us now turn to the interpretation of the electric flux tube solution of Sec. II. For convenience let us compactify the direction along which the electric flux points, so that the fundamental string charge associated with the electric flux has the simple interpretation of a fundamental string winding number. In analogy with the rolling tachyon solution, we should expect that the classical results will be a good approximation to the complete answers when the string coupling  $g$  is small, and the energy density and the winding number density are of order  $g^{-1}$ . This is large for small  $g$ . In this case we could interpret the freedom of spreading out the winding number simply to the possibility of distributing these large numbers of fundamental strings arbitrarily in the hyperplane transverse to the compact direction. The classical dynamics of the tachyon effective field theory then describes the time evolution of the expectation values of various physical quantities for such a system.

The analysis of Sec. IV shows that in the presence of such an electric flux, a D- $(p-1)$ -brane placed transverse to the flux will fatten with a width of order  $|\vec{\Pi}|/\tilde{T}_p$ . We could ask if there is a physical understanding of this fattening based on our interpretation given above. It is tempting to suggest that this fattening is caused by a large number of fundamental strings ending on the D- $(p-1)$ -brane from both sides. Since the fundamental string deforms the D-brane into the shape of a spike as described in Sec. V, the average effect of a large density of spikes on both sides of the D-brane will be to

effectively fatten the D-brane. Since the total number of spikes is proportional to the total flux  $|\vec{\Pi}|$ , this will easily explain why the excess energy density away from the original plane of the D- $(p-1)$ -brane is proportional to  $|\vec{\Pi}|$ .

## VII. SUMMARY

We conclude the paper by summarizing the main results.

- (1) The tachyon effective field theory is known to contain electric flux tube solutions for which the electric field is at its critical value and the tachyon is at infinity. These flux tubes have many properties in common with fundamental strings. We show that if such a flux tube “ends” on a kink solution of the effective field theory representing a BPS D- $(p-1)$ -brane, then the tachyon cannot increase faster than  $x^{1/2}$  as we move a distance  $x$  away from the plane of the kink along the flux tube. Thus the flux tube approaches its asymptotic configuration, where the tachyon is at its vacuum value  $\infty$ , very slowly. There is no explicit classical solution known at present describing such configurations, but we have argued that even if we are able to construct such solutions, they will be missing out on one important property of the fundamental string—that of the exchange interaction.
- (2) We propose an alternative form of the electric flux tube solution for which the tachyon is finite in the region which carries the electric flux. Energetic considerations then force this region to have zero volume. While this is an improvement over the electric flux tube solutions mentioned in item (1), this constraint is not sufficiently strong to confine the electric flux to a one-dimensional subspace, as is required if it has to describe a single fundamental string. We suggest that higher-derivative and quantum corrections, which are expected to be significant for these solutions, could be responsible for localizing the fundamental string charge to a one-dimensional subspace.
- (3) A recent analysis of time-dependent solutions describing the decay of a non-BPS D-brane has suggested that during the decay process all the energy of the D-brane is converted to closed strings. We suggest that this is an alternative description of the phenomena that we see in the analysis in open string theory and verify this by comparing the properties of the tachyon matter obtained in the classical open string analysis with the properties of the system of closed strings expected to be produced during the decay process. This analysis suggests that tachyon matter in effect describes a system of closed strings at high energy density. In the same spirit, we also suggest that the electric flux tube solutions described in item (1) represent a system of closed strings at high energy density and high winding charge density.

## ACKNOWLEDGMENTS

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